Stocastic Sec 3

Sheet 2

(13) Prove $B - A = A^c \cap B$

متروك للأسبوع القادم

The Conditional Probability of an event A given that B has occured is denoted by $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Sheet 3

[4] 25 % faild in math $\to E_1$

15 % " chemistry $\to E_2$

10 % " both $\to E_1 \cap E_2$

a) If he faild in chemistry, what is the probability that he falled in math

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{0.1}{0.15}$$

b) If he faild in math, what is the probability that he faild in chemistry

$$P(E_2|E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{0.1}{0.25}$$

c) faild in math or chemistry

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= 0.25 + 0.15 - 0.1$$
$$= 0.3$$

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[5] $P(A) = 0.5$ $P(B) = 0.33$

$P(A \cap B) = 0.25$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.25}{0.5}$$

$$P(A|B) = \frac{P(B \cap A)}{P(B)} = \frac{0.25}{0.33}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{7}{12}$$

$$P(A^c|B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{1 - P(A \cup B)}{1 - P(B)}$$

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[6] $P(A) = \frac{3}{8}$ $P(B) = \frac{5}{8}$, $P(A \cup B) = \frac{3}{4}$

a) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

b) $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

Independent Events:

$$P(B) = P(B|A) = \frac{P(B \cap A)}{P(A}$$

$$P(A \cap B) = P(A)\,P(B)$$

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[1] A, B are Independent events

Prove that

a) A^c and B^c are independent

$P(A^c \cap B^c) = P(A^c)\,P(B^c) \to$ required

$$P(A^c \cap B^c) = 1 - P(A \cup B)$$
$$= 1 - P(A) - P(B) + \underbrace{P(A \cap B)}_{P(A)P(B)}$$
$$= 1 - P(A) - P(B) + P(A)\,P(B)$$
$$= [1 - P(A)] - P(B)\,[1 - P(A)]$$
$$= [1 - P(A)] * [1 - P(B)]$$
$$= P(A^c)\,P(B^c)$$

$\therefore$ A^c and B^c are Independent events

b) A and B^c are Independent

$P(A \cap B^c) = P(A)\,P(B^c) \longrightarrow$ required

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[2] 3 Fair Coins Find probability that they are all heads if $\to A$

a) the first coin is head $\to B$

b) one of coins is head $\to C$

$P(A) = 1/8$

$P(B) = 1/2$ $P(C) = 7/8$

a) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/8}{1/2} = \dfrac{2}{8} = \dfrac{1}{4}$

b) $P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{1/8}{7/8} = \dfrac{1}{7}$

$\boxed{10}$ Probability of a man lives 10 more years is $\frac{1}{4}$, Probability of his wife is $\frac{1}{3}$

a) $E_1 \rightarrow$ man lives 10 more $\rightarrow P(E_1) = \frac{1}{4}$

$E_2 \rightarrow$ wife 〃 〃〃 $\rightarrow P(E_2) = \frac{1}{3}$

$$P(E_1 \cap E_2) = P(E_1)\, P(E_2)$$
$$= \frac{1}{4} * \frac{1}{3}$$

where E_1, E_2 are independent

b) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$
$$= \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$$

c) $P(E_1 \cup E_2)^c = 1 - P(E_1 \cup E_2)$

d) $P(E_2 - E_1) = P(E_2 \cap E_1^c)$

$\Rightarrow P(E_2) - P(E_1 \cap E_2)$ other solution

$\boxed{11}$ $A \rightarrow$ Famly has children from both gender

$B \rightarrow$ Famly has at most one boy

i) show that A and B are ~~[illegible]~~ Independent if Famly has 3 child

ii) 〃〃 〃〃〃〃〃〃〃 dependent 〃 〃 〃 2 child

i) Sample Space

$S = \{bbb, bbg, bgb, gbb, bgg, gbg, ggb, ggg\}$

$P(A) = 6/8$

$P(B) = 4/8$

$P(A \cap B) = 3/8$

$$P(A) \cdot P(B) = \frac{6}{8} \cdot \frac{4}{8} = \frac{3}{8}$$

$\because P(A \cap B) = P(A)\, P(B) = \frac{3}{8}$

$\therefore A, B$ are independent

ii) $S = \{bb, bg, gb, gg\}$

$P(A) = 1/2$

$P(B) = 3/4$

$P(A \cap B) = \frac{1}{2}$

$P(A) . P(B) = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$

$P(A \cap B) \neq P(A)\, P(B)$

A, B are dependent

Report

3, 8, 12, 13, 17